32.16 **$500,000\frac{lb_m}{hr}$ of $500°F$ saturated water enters a steam boiler and exits as 97% quality steam at $1400psia$. The heat is supplied by flue gas which surrounds a bank of $20,000$ tubes carrying the water. Each tube has a surface area of $4ft^2$, an overall heat transfer coefficient of $250\frac{Btu}{hr \cdot ft^{2} \cdot °F}$, and a temperature differential of $25°F$. What is the efficiency of the boiler?**

A. 55%

B. 62%

C. 67%

D. 73%

Consider the entering water as State 1 and the leaving saturated mixture as State 2. Use the Properties of Saturated Water and Steam table to obtain the enthalpy at State 1.

$$T_1 = 500°F\,(saturated)$$

$$h_1 = h_f = 488\frac{Btu}{lb}$$

Use the steam table again to obtain the enthalpy values h_f and h_{fg} at $1400psia$, and use the quality to determine the enthalpy at State 2.

$$P_2 = 1400psia$$

$$h_f = 598.9\frac{Btu}{lb}$$

$$h_{fg} = 575.6\frac{Btu}{lb}$$

$$h_2 = h_f + \chi h_{fg} = 598.9\frac{Btu}{lb} + (0.97)\left(575.6\frac{Btu}{lb}\right) = 1157.2\frac{Btu}{lb}$$

Calculate the heat added to the steam.

$$\dot{Q}_{steam} = \dot{m}\Delta h = \dot{m}\,(h_2 - h_1)$$

$$\dot{Q}_{steam} = \left(500,000\frac{lb}{hr}\right)\left(1157.2\frac{Btu}{lb} - 488\frac{Btu}{lb}\right) = 3.35 \times 10^{8}\frac{Btu}{hr}$$

The heat delivered by the flue gas is based on the overall heat heat transfer, depending on surface area, overall heat transfer coefficient, and delta T. The surface area is a function of the number of tubes times the surface area for each tube. Calculate the surface area and the input heat.

$$A = (20,000)\left(4ft^2\right) = 80,000ft^2$$

$$\dot{Q}_{flue\,gas} = UA\Delta T$$

$$\dot{Q}_{flue\,gas} = \left(250\frac{Btu}{hr \cdot ft^2 \cdot {}^\circ F}\right)\left(80,000ft^2\right)(25^\circ F) = 5 \times 10^8 \frac{Btu}{hr}$$

The efficiency of the boiler is the output divided by the input.

$$\eta = \frac{\dot{Q}_{steam}}{\dot{Q}_{flue\,gas}} = \frac{3.35 \times 10^8 \frac{Btu}{hr}}{5 \times 10^8 \frac{Btu}{hr}} = 0.67$$

Answer C